Problem 1: Conditional probability (from Ewens & Grant Ch. 1)

Two fair coins are tossed and you are told that at least one coin came up heads. What is the probability that both coins came up heads? Note: the two “instinctive” answers $\frac{1}{2}$ and $\frac{1}{4}$ are both incorrect. It is necessary to use the formula for conditional probability.

Problem 2: Weather Markov chain

Recall the Markov chain presented in the lecture, representing the weather on day $n$. It consists of three states corresponding to (1) sunny, (2) cloudy, and (3) rainy weather. The transition probability matrix is

$$P = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.2 & 0.7 \end{bmatrix}.$$

a) Given that the weather is sunny on Monday, what is the probability that it is rainy on Tuesday?

b) Given that the weather is sunny on Monday, what is the probability that it is rainy on Wednesday?

c) Given that the weather is sunny on Monday and rainy on Tuesday, what is the probability that it is rainy on Wednesday?

d) Use the method of Section 4.7.2 in Ewens & Grant to find the stationary distribution for this Markov chain.

e) Compute $P^2$, $P^4$, $P^8$ and $P^{16}$ (preferably using a computer or a suitable calculator). Comment the result, comparing with the stationary distribution.

Problems from Ewens & Grant

4.4, 4.5, 4.6, 4.8, 10.5